

● Specific heat: The amount of heat needed to rise the temperature of one gram of a substance by one degree, usually measured in joules per kilogram per kelvin. (13)

● Two specific heats of a gas:

When a gas is heated, ordinarily there is an increase in volume as well as pressure in addition to the rise of temperature. Either the volume or the pressure may be kept constant. Therefore a gas has two specific heats.

- (i) specific heat at constant volume, and
- (ii) specific heat at constant pressure.

If dQ is the amount of heat required to rise the temperature of a unit mass of the gas through $dT^\circ C$, keeping the volume constant, then, specific heat at constant volume $C_v = \left(\frac{dQ}{dT}\right)_v$.

If dQ is the amount of heat required to rise the temperature of a unit mass of the gas through $dT^\circ C$ keeping the pressure constant, then, specific heat at constant pressure $C_p = \left(\frac{dQ}{dT}\right)_p$.

● Difference between two specific heats:

$$C_v = \left(\frac{\partial Q}{\partial T}\right)_v$$

From 1st law of thermodynamics,

$$dQ = dU + PdV$$

$$\text{since, } v = \text{const, so, } dV = 0 \quad \left(\frac{\partial Q}{\partial T}\right)_v = \left(\frac{\partial U}{\partial T}\right)_v$$

$$C_v = \left(\frac{\partial U}{\partial T}\right)_v \quad \text{--- (1)}$$

$$\text{or, } \boxed{dU = C_v dT}$$

$$\text{Again } C_p = \left(\frac{\partial Q}{\partial T}\right)_p \Rightarrow$$

$$\text{From, } dQ = dU + PdV \Rightarrow \left(\frac{\partial Q}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + P \left(\frac{\partial V}{\partial T}\right)_p$$

$$C_p = \left(\frac{\partial U}{\partial T}\right)_p + P \left(\frac{\partial V}{\partial T}\right)_p \quad \text{--- (2)}$$

$$C_p - C_v = \left(\frac{\partial U}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v = \left(\frac{\partial U}{\partial T}\right)_p + P \left(\frac{\partial V}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v \quad \text{--- (3)}$$

$$\text{Again } U = U(v, T)$$

$$\text{Diff } \partial \text{ partially, } dU = \left(\frac{\partial U}{\partial v}\right)_T dv + \left(\frac{\partial U}{\partial T}\right)_v dT \quad \text{--- (4)}$$

$$\Rightarrow \left(\frac{\partial U}{\partial T}\right)_p = \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p + \left(\frac{\partial U}{\partial T}\right)_v$$

Putting the value of eq (4) in eq (3)

$$C_p - C_v = \left(\frac{\partial U}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p + \left(\frac{\partial U}{\partial T}\right)_v + P \left(\frac{\partial v}{\partial T}\right)_p - \left(\frac{\partial U}{\partial T}\right)_v$$

$$\boxed{C_p - C_v = \left[\left(\frac{\partial U}{\partial v}\right)_T + P \right] \left(\frac{\partial v}{\partial T}\right)_p}$$

Since, there is no intermolecular attraction, its internal energy is wholly kinetic and remains unchanged in free expansion.

Thus, $\left(\frac{\partial U}{\partial T}\right)_T = 0$

$\therefore C_p - C_v = P \left(\frac{\partial v}{\partial T}\right)_P$

For 1 mole of ideal gas, $Pv = RT \Rightarrow v = \frac{RT}{P}$

$\left(\frac{\partial v}{\partial T}\right)_P = R/P$

$\therefore C_p - C_v = P \cdot R/P$

$C_p - C_v = R$

For n mole, $C_p - C_v = nR$

This is known as Mayer's relation

● Calculate the amount of work done during an isothermal expansion:

Isothermal change (Process) : when a change in pressure and volume of a substance take place but the temperature remains constant, the change is said to be isothermal.

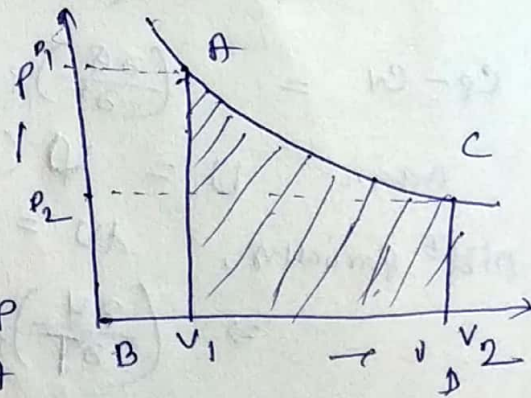
Thus in an isothermal change in temperature is kept constant by adding heat or taking it away from the substance. As there is no change in temperature, there is no change in internal energy i.e., $dU = 0$.

According to 1st law, $dQ = dU + PdV \Rightarrow dQ = PdV$

For a perfect gas an isothermal change is represented by Boyle's law,

$Pv = \text{const}$

Work done : consider a gram molecule of a perfect gas contained in a cylinder having perfect conducting bottom fitted with a piston. Let V_1 be the volume, P be the pressure and T_K the temperature of the gas. The cylinder is placed on a source of heat at the same temperature T_K .



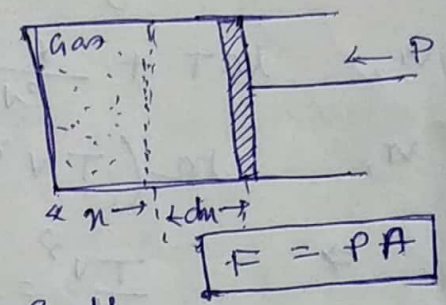
If the piston is now allowed to move slowly outwards the gas expands, does some work and due to it the temperature tends to fall. Heat is, therefore, absorbed from the source at the constant temperature T and thus the temperature of the gas is kept the same. The expansion is isothermal.

If A is the area of cross-section of the piston, and it moves through a small distance dx so that the change in volume dV , then the small amount of work done by the gas is given by,

$$dw = P A dx = P dV$$

For ideal gas, $PV = RT \Rightarrow P = \frac{RT}{V}$

$$dw = \frac{RT}{V} dV$$



Hence the total work done when the gas expands isothermally from a volume V_1 to the volume V_2 is given by the area ABCD included between the isothermal AB and the ordinates AC and BD corresponding to the volumes V_1 and V_2 .

$$\therefore \text{total work done } w = \int_{V_1}^{V_2} \frac{RT}{V} dV = RT \ln \frac{V_2}{V_1}$$

For n moles of a perfect gas
$$W = nRT \ln \frac{V_2}{V_1}$$

Calculate the amount of work done during an adiabatic expansion:

Adiabatic change: when a change in pressure and volume of a substance take place but no heat is allowed to enter or leave it, the change is said to be adiabatic.

Hence in an adiabatic change, the temperature does not remain constant and no heat from outside is supplied to the system or taken away from it.

During an adiabatic process no heat enters or leaves the gas

$$\text{i.e. } dq = 0$$